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LETTER TO THE EDITOR

Local magnetisation of the n-vector model with a large-scale defect in the limit $n \to \infty$

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Abstract. The local magnetisation of the three-dimensional n-vector model is studied near the large-scale d'-dimensional defect in the limit $n \to \infty$. The model is characterised by the fact that the spin lengths close to the defect are changed as $S_r^2 = n(1-\lambda/r)$ with the distance r from the defect. It is shown that near the critical point the local magnetisation exhibits a non-universal behaviour if $d' = 0, 1$ and a non-scaling behaviour if $d' = 2$. In the first case the critical exponents β' and ν' are calculated up to the order of λ and it is shown that these exponents satisfy the usual scaling law relations up to this order.

In the previous letter (Bariev and Ilaldinov 1989) the effect of the large-scale d'-dimensional defect on the correlation function of the three-dimensional n-vector model was studied in the limit $n \to \infty$. We considered the model in which the defect changes the spin lengths in its vicinity. These changes decay as $-\lambda/r$ with the distance r from the defect

$$S_j^2 = n(1-\lambda/r_j)$$
$$r_j = [j_{d'+1}^2 + \ldots + j_3^2 + 1]^{1/2}. \tag{1}$$

It was shown that on the defect plane the correlation function exhibits a non-scaling behaviour if $d' = 2$ and non-universal behaviour if $d' = 1$. In this letter we shall consider the local magnetisation in the centre of the defect for the different values $d' = 0, 1, 2$.

The partition function of the model under consideration in an inhomogeneous field h_j is

$$Z_h = \int_{-\infty}^{\infty} \prod_{j,k} d\sigma_m(m) \exp\left(\frac{1}{2}\sum_{j,k} K_{jk}\sigma_j(m)\sigma_k(m) + \sum_j h_j\sigma_j\right)$$
$$\times \prod_j \delta\left(n(1-\lambda/r_j) - \sum_m \sigma_j^2(m)\right) \tag{2}$$

where the interaction constants $J_{jk} = kTK_{jk}$ are unequal to zero only for the nearest neighbours.

The local magnetisation at the jth site of lattice is defined by

$$M_j \overset{\text{def}}{=} \frac{1}{n}\lim_{h_j \to 0} \frac{\partial \ln Z_h}{\partial h_j}. \tag{3}$$

Using the procedure of Abe (1981) we can reduce the calculation of the correlation function in the non-zero field to the following mathematical problem:

$$G(j,k) = (A^{-1})_{jk} \qquad A_{jk} = 2t_j^0\delta_{jk} - K_{jk}. \tag{4}$$

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The coordinates of the saddle point t_j^0 are determined from the self-consistency condition

$$G(j,j)+M_j^2=1-\lambda/r_j. \tag{5}$$

Unlike the previous letter (Bariev and Ilaldinov 1989) where the analogous problem was considered for the critical temperature $T=T_c$ the problem (3)-(5) is considered for the temperature $T\leq T_c$. In particular for the defect-free lattice we have the following expression for the magnetisation (Abe 1981):

$$M_0=[(K-K_c)/K_c]^{1/2}. \tag{6}$$

For the model with large-scale defect we obtained the following results:

$$M=M_0\left(1-\frac{8K_c\lambda}{\pi}-(\ln^2\alpha-\ln^2\Lambda-\ln\alpha+\ln\Lambda+\pi^2/6)+\mathrm{O}(\lambda^2)\right) \tag{7}$$

if $d'=2$ and

$$M=M_0[1+x_1^{d'}\lambda(\ln\alpha-\ln\Lambda)+\mathrm{O}(\lambda^2)] \tag{8}$$

if $d'=1,0$ where

$$x_1^1=8K_c \qquad x_1^0=16K_c/\pi \qquad \alpha=16(K-K_c)$$

and Λ is the cutoff parameter.

Then by means of a standard exponentiation we have

$$M=M_0[1+\lambda A_1+\mathrm{O}(\lambda^2)]\alpha^{-(8\lambda K_c/\pi)(\ln\alpha-1)+\mathrm{O}(\lambda^2)} \qquad \text{if } d'=2 \tag{9}$$

and

$$M=M_0[1+\lambda B_1^{d'}+\mathrm{O}(\lambda^2)]\alpha^{\lambda x_1^{d'}+\mathrm{O}(\lambda^2)} \qquad \text{if } d'=1,0 \tag{10}$$

where

$$A_1\frac{8K_c}{\pi}[\ln^2\Lambda-\ln\Lambda-\pi^2/16] \qquad B_1^1=-8K_c\ln\Lambda \qquad B_1^0=-\frac{16K_c}{\pi}\ln\Lambda.$$

The result (9) means a breakdown of scaling since the magnetisation has a non-power dependence on (T_c-T). The appearance of the term $M_0\ln^2\alpha$ leads to such a conclusion. The result (10) means non-universal critical behaviour of magnetisation near the defect. The non-universal behaviour manifests itself in the fact that the critical exponent β' depends on the microscopic parameter λ $(\beta'=\frac{1}{2}+\lambda x_1^{d'}+\mathrm{O}(\lambda^2))$†. The appearance of the term $M_0\ln\alpha$ in (8) means the fulfilment of the necessary condition for it (Kadanoff and Wegner 1971). Using (10) and the result of the previous letter $(\eta'=16\lambda K_c)$ it is easy to verify that in the case $d'=1$ the critical exponents β' and η' obey the usual scaling relation $(d-2+\eta')\nu'=2\beta'$ with $\nu=1$ and $d=3$.

It should be noted that a simple-minded exponentiation of the logarithmic term sometimes leads to a misleading result. Therefore it is necessary to study the higher-order terms of expansion of the correlation function in powers of λ. Consideration of this problem as well as details of calculation will be published elsewhere. In

† The calculation of the correlation functions up to λ^2 confirms these conclusions. Details of the calculations will be published elsewhere.

conclusion we note that the results (9) and (10) are in agreement with the predictions of the phenomenological approach (Bariev 1988).

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